Non-degenerate semiconductors

Non-degenerate semiconductor ⇒ the Fermi level lies within the bandgap, and is generally close to the mid-gap

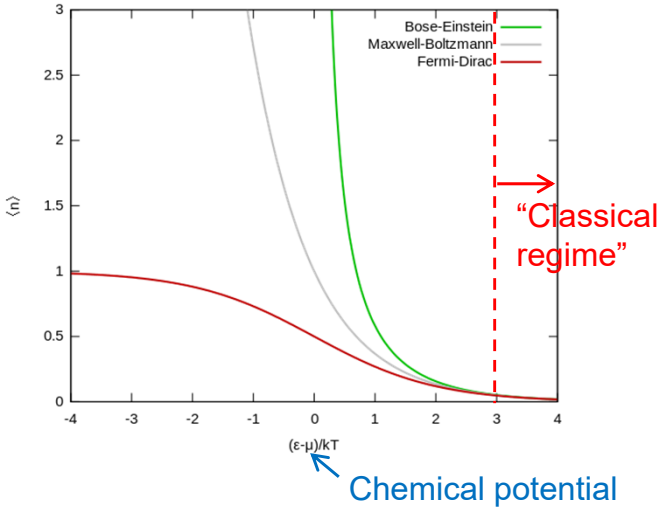
Then it comes $|E-E_F| >> k_B T$ (300 K: $k_B T \approx 25$ meV to be compared to $E_g/2 > 500$ meV (see, e.g., the case of Si, GaAs, GaN, etc.))

⇒ **Boltzmann approximation** (i.e., the carrier number is low enough so that Pauli exclusion principle does not apply). The occupancy statistics becomes:

$$f_c(E) = \frac{1}{1+e^{(E-E_F)/k_B T}} \Rightarrow f_c(E) \approx e^{-(E-E_F)/k_B T}$$

$$f_v(E) = 1 - f_c(E) \approx e^{-(E_F-E)/k_B T}$$

Occupancy statistics



Non-degenerate semiconductors

One then integrates using $\rho_{3D}(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)\sqrt{E-E_0}$ **3D-DOS per energy unit**

$$n_c = \int_{E_c}^{+\infty} \rho_c(E) f_c(E) dE \quad \Rightarrow \quad n = N_c e^{-(E_C - E_F)/k_B T}$$

$$n_v = \int_{-\infty}^{E_v} \rho_v(E) f_v(E) dE \quad \Rightarrow \quad p = N_v e^{-(E_F - E_V)/k_B T}$$

$$N_{c(v)} = 2\left(\frac{2\pi m^* k_B T}{h^2}\right)^{3/2} = cst \cdot \left[\frac{m^*}{m_0} T\right]^{3/2}$$

$$= 2.5\times 10^{19}\left(\frac{m^*}{m_0}\right)^{3/2}\left(\frac{T}{300}\right)^{3/2} \text{ cm}^{-3}$$

$N_{c(v)}$ are the effective density of states

Non-degenerate semiconductors

Effective density of states $\Rightarrow$ a band can be described by a discrete level with a concentration N_c and filled with a probability $\exp[-(E_c-E_F)/k_BT]$

Effective density of states ($N_{c(v)}$) *at 300 K for different semiconductors*

	N_C (10^{19} cm^{-3})	N_V (10^{19} cm^{-3})
Si	2.8	1.0
Ge	1	0.4
GaAs	0.04	1.2

Fermi level calculation

$$n = N_c e^{(E_F - E_c)/k_B T}$$

$$p = N_v e^{(E_v - E_F)/k_B T}$$

Remark: n and p can be experimentally measured (Hall effect, electrochemical *C-V* profiling)

$$E_F = E_c - k_B T \ln \frac{N_c}{n} = E_v + k_B T \ln \frac{N_v}{p}$$

- When n (*or* p) << N_c (*or* N_v) $\Rightarrow$ E_F lies in the bandgap
- When n (*or* p) > N_c (*or* N_v) $\Rightarrow$ the Fermi level lies within the CB (or VB)
  $\Rightarrow$ The semiconductor is then said to be ***degenerate***

Thermodynamic equilibrium

At equilibrium ⇒ same chemical potential, same E_F, across the sample whatever the semiconductor

The product *np* for a non-degenerate semiconductor is then independent of the Fermi level position and is given by:

$$np = N_c N_v e^{-\frac{E_c - E_v}{k_B T}} = N_c N_v e^{-\frac{E_g}{k_B T}}$$

Product that depends on $m^{*3/2}$, $T^{3/2}$, and E_g

For a given semiconductor, ***np* is a function of temperature**. This is a **mass action law**, which expresses the thermodynamic equilibrium condition for electrons and holes

Intrinsic semiconductors

A pure and perfect semiconductor is intrinsic

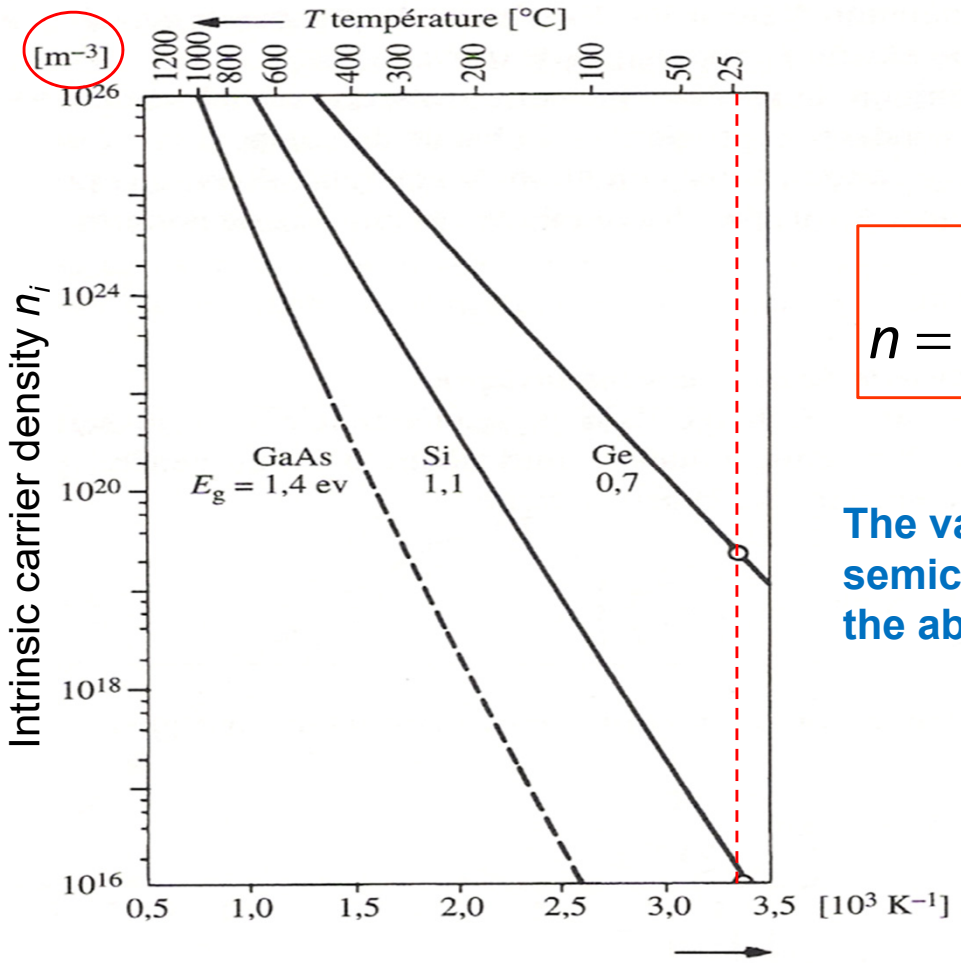
The origin of carriers present in the CB and VB is endogenous, i.e., free carriers are only due to the thermal activation process of electrons from the VB to the CB

The condition for electrical neutrality throughout the crystal leads to $n = p = n_i$ at equilibrium, so that:

$$np = n_i^2 = N_c N_v e^{-\frac{E_c - E_v}{k_B T}} = N_c N_v e^{-\frac{E_g}{k_B T}}$$

$$n = p = n_i = \sqrt{N_c N_v}\, e^{-\frac{E_g}{2k_B T}}$$

Intrinsic semiconductors



$$n = p = n_i = \sqrt{N_c N_v}\, e^{-\frac{E_g}{2k_B T}}$$

The value of n_i will set the sensitivity of a given semiconductor to residual impurities and hence the ability to precisely control *n*- and *p*-type doping

Intrinsic semiconductors

Fermi level position in an intrinsic semiconductor

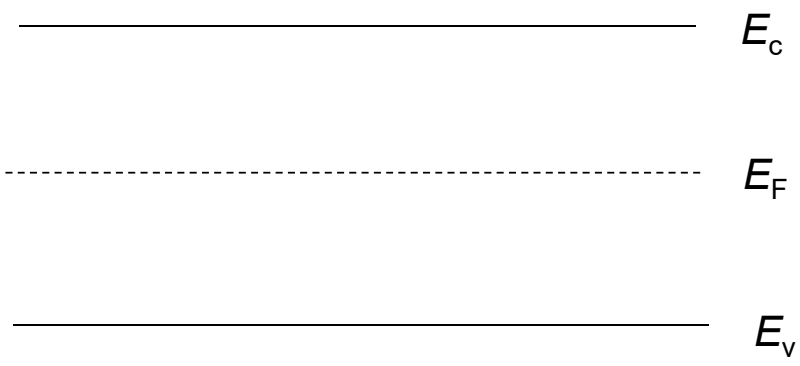
$$n = N_c e^{(E_F - E_c)/k_B T}$$

$$p = N_v e^{(E_v - E_F)/k_B T}$$

$$E_F = E_c - k_B T \ln \frac{N_c}{n} = E_v + k_B T \ln \frac{N_v}{p}$$

$$E_F = \left(\frac{E_v + E_c}{2} \right) + \frac{k_B T}{2} \ln \left(\frac{N_v}{N_c} \right)$$

N_v and N_c are comparable therefore **E_F is close to the mid-gap**



Semiconductor – degenerate case

Degenerate SC ⇒ highly doped

$$E_F = E_c - k_B T \ln \frac{N_c}{n}$$ with $n > N_c$ (case of an n-type SC)

The Fermi level lies within the CB ⇒ Boltzmann approximation is **no longer valid** (cf. slide #4)

One may consider as a rough approximation a Heaviside step function to account for the occupancy statistics:

$f(E) = 1$ when $E < E_F$
$f(E) = 0$ when $E > E_F$

$$n = \int_{E_c}^{E_F} \rho_c(E)\,dE = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} \int_{E_c}^{E_F} (E - E_c)^{1/2}\,dE$$

$$= \frac{1}{3\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} (E_F - E_c)^{3/2}$$

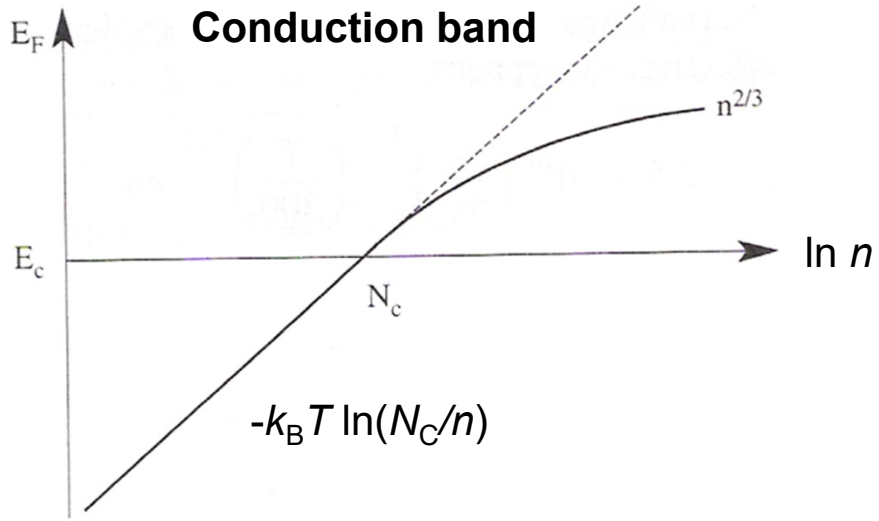
n* is independent of *T

More detailed description to be seen in the exercises !

Semiconductor – degenerate case

A degenerate SC behaves like a metal (but this is not exactly a metal, why?)

We speak about a semimetallic behavior



For $n > N_c$, the Fermi level position varies as $n^{2/3}$

The $np = n_i^2$ relationship is not valid anymore in the degenerate case!